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Theory of magneto-oscillation effects in quasi-two-dimensional semiconductor structures

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Abstract

A theory of magneto-oscillations of kinetic coefficients is developed for two-dimensional structures with several occupied size-quantized subbands. The Green function of a multilevel system is calculated, making allowance for intersubband scattering in moderate magnetic fields at both zero and non-zero temperatures. The temperature dependence of the Shubnikov–de Haas effect is investigated. It is shown that oscillations with frequency proportional to the concentration in the upper subband appear even if occupation of this subband is low. For the first time, the amplitude of the conductivity magneto-oscillations that are not thermally damped out is calculated. It is shown that they have frequencies proportional to the differences of the energies of size-quantization. It is predicted that there will be emergence and disappearance of beats in the magnetoconductivity at relatively small occupation of the upper subband.

1. Introduction

Thermodynamic and kinetic coefficients like the heat capacity, magnetic susceptibility, conductivity, thermal conductivity, and ultrasonic absorption are well known to oscillate in a magnetic field in structures with a degenerate electron gas at low temperatures. A common reason for all of these effects is the consecutive crossing of the Fermi level by Landau levels in a quantized magnetic field. The frequency of the oscillations in a reciprocal magnetic field is proportional to the Fermi energy.

The magnetic field regions in which these oscillations are observed differ strongly between the two-dimensional (2D) and 3D cases. In the bulk, the oscillations appear in a 'classically strong' magnetic field, when $\omega_c\tau \gg 1$. Here, ω_c is the cyclotron frequency and τ is the carrier relaxation time. In contrast, in 2D structures, this effect appears in a moderate magnetic field, when the change of the density of states due to the magnetic field is small [1, 2]. This takes place when $\exp(-\pi/\omega_c\tau) \ll 1$, i.e. at $\omega_c\tau \lesssim 1$. Therefore in contrast to the bulk case, where the oscillations contain some multiple of the main harmonic, the rigorous theoretical treatment gives oscillations with only one frequency for ultraquantum 2D systems. The corresponding theory was developed in reference [1].

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A qualitatively new situation arises in quasi-two-dimensional (quasi-2D) structures, where two or several size-quantized subbands are occupied. In this case, each subband may give rise to an oscillation with its own period. The oscillation amplitudes are determined by the total scattering probability including intersubband transitions.

The temperature behaviour of the oscillation effects in quasi-2D systems is also unexpected. At one-subband occupation, the oscillations damp out with temperature because of the carrier distribution broadening. If two or more subbands are occupied, the oscillations appear at differential frequencies which are proportional to the energy distances between the size-quantized levels. These are harmonics, which are conserved as temperature increases [3].

The resistivity oscillations in a magnetic field—the Shubnikov–de Haas effect—were observed experimentally in quasi-2D systems in references [4–6]. The case of two subbands was studied theoretically in references [4, 7]. However, the oscillation terms were taken into account inconsistently, and, for this reason, the analysis of the experimental data [4, 5] was carried out incorrectly. In reference [7], only one conductivity tensor component was considered and its temperature dependence was calculated inexactly.

Oscillations with several periods were observed in multivalley bulk semiconductors for intense intervalley scattering [8, 9]. However, it is impossible to apply these results to quasi-2D systems because the parameter $\omega_c\tau$ is not large, in contrast to the 3D case. For the same reason, the density matrix method used for the 3D case fails. Instead, we will apply the Green function method.

The Green function of a quasi-2D system in a quantized magnetic field will be obtained at zero and non-zero temperatures. With its help, we will study the conductivity oscillations in a magnetic field. The aim of this paper is to calculate the magnetoconductivity tensor for a quasi-2D electron system at non-zero temperature. The theory will be developed for an arbitrary number of size-quantized subbands, and the case of two subbands will be considered in detail. Spin effects will be neglected for simplicity.

In real 2D structures, scattering may be governed by different factors. The scattering from remote impurities may be most important, or the scattering from interface roughness may be dominant, or, at a slightly higher temperature, the scattering from acoustic phonons may play the crucial role [10]. However, in the present paper we consider a multisubband system and investigate the effect of intersubband scattering on the transport properties of such a system. Under transitions from one subband to another, a large momentum having the order of the Fermi momentum is transferred to the scatterers. Hence the characteristic scattering length has to be of the order of the Fermi wavelength, i.e. the scattering potential has to be short range [7, 11]. This case is considered in this paper.

The paper is organized as follows. In section 2, the system of Dyson equations is solved and the Green functions are obtained at zero and non-zero temperatures. In section 3, the magnetoconductivity tensor is calculated for a quasi-2D system. In section 4, we consider the magnetoresistance for the case of two filled subbands in detail and perform a qualitative comparison with the available experimental data. In the concluding section, the main results of the paper are presented. In the appendices, some details of the calculations are given.

2. The Green function of a quasi-2D system in a magnetic field

2.1. Zero temperature

In the general case, the Green function of non-interacting electrons in a magnetic field at zero temperature may be presented as follows:

$$G_\varepsilon(\boldsymbol{r},\boldsymbol{r}') = \sum_{j,j'}\sum_{n,k_y} G_{jj'}(\varepsilon;n,k_y)\psi_{jnk_y}(\boldsymbol{r})\psi^*_{j'nk_y}(\boldsymbol{r}'). \tag{1}$$

Here $\psi_{jnk_y}(\boldsymbol{r})$ are the electron wavefunctions in a heterostructure under a perpendicular magnetic field in the Landau gauge, j and j' are numbers labelling size-quantized subbands, n and k_y are numbers labelling Landau levels and values of wavevectors in the plane of a heterostructure, respectively.

We assume that there are many Landau levels under the chemical potential in each subband:

$$\mu_j/\hbar\omega_c \gg 1 \tag{2}$$

and that the condition for a 'good conductor' is fulfilled for all subbands:

$$\mu_j\tau_j/\hbar \gg 1. \tag{3}$$

Here μ_j is the energy distance between the chemical potential and the bottom of the jth subband at zero field, and τ_j is the relaxation time of carriers in the jth subband.

The coefficients $G_{jj'}(\varepsilon;n,k_y)$ are determined from the system of M^2 Dyson equations, where M is the number of filled subbands. We assume the energy distances between subbands to be large:

$$|\mu_j - \mu_{j'}| \gg \hbar/\tau_j. \tag{4}$$

Hence the non-diagonal coefficients, $G_{jj'}$, are much smaller than the diagonal ones:

$$G_\varepsilon(\boldsymbol{r},\boldsymbol{r}') = \sum_{j=1}^{M}\sum_{nk_y} G_j(\varepsilon;n,k_y)\psi_{jnk_y}(\boldsymbol{r})\psi^*_{jnk_y}(\boldsymbol{r}'). \tag{5}$$

We consider that scattering takes place from a number of randomly distributed short-range potentials. In this approximation the self-energy parts are independent of the Landau-level index, n. This has been demonstrated for a 3D system [12], for a 2D system with one occupied subband [13], and, similarly, for a two-subband system [7]. Therefore the coefficients G_j in (5) have the following form:

$$G_j(\varepsilon,\xi_{j,n}) = [\varepsilon - \xi_{j,n} - X_j(\varepsilon)]^{-1} \tag{6}$$

where $\xi_{j,n}$ is the energy distance between Landau level n in the jth subband and the chemical potential:

$$\xi_{j,n} = \hbar\omega_c(n+1/2) - \mu_j. \tag{7}$$

The self-energy parts, X_j, are determined in the framework of the self-consistent Born approximation from the system of equations

$$X_j(\varepsilon) = \frac{\hbar\omega_c}{\pi}\sum_n\sum_{j'=1}^{M}\frac{\hbar}{2\tau_{jj'}}G_{j'}(\varepsilon,\xi_{j',n}). \tag{8}$$

This system (8) was obtained in reference [7] for two-subband filling. Here τ_{jj} and $\tau_{jj'}$ $(j \neq j')$ are the intrasubband and intersubband scattering times at zero magnetic field, respectively. Note that the scattering time $\tau_{jj'}$ for the intersubband transition from the jth to the j'th subband coincides with the time for the reciprocal process, $\tau_{j'j}$, due the effective masses being the same in all subbands. The total relaxation times in subbands in the absence of a magnetic field, τ_j, are given by

$$\frac{1}{\tau_j} = \sum_{j'=1}^{M}\frac{1}{\tau_{jj'}}. \tag{9}$$

In the presence of a quantized magnetic field the relaxation times depend on the field. They are defined by the imaginary parts of the self-energy $X_j(\varepsilon)$ and can be obtained from the equation system (8). In order to solve this system one uses the Poisson summation formula

$$\sum_{n=0}^{\infty} f(n) = \sum_{k=-\infty}^{\infty} \int_0^{\infty} \mathrm{d}n \ \exp(2\pi \mathrm{i}kn) f(n) + \frac{f(0)}{2}. \tag{10}$$

Since below we have to treat Green functions at $|\varepsilon| \ll \mu_j$, we neglect the last term in (10) and obtain

$$X_j(\varepsilon) = \sum_{j'=1}^{M} \frac{\hbar}{2\pi\tau_{jj'}} \sum_{k=-\infty}^{\infty} \int_{-\mu_{j'}}^{\infty} \mathrm{d}\xi \left\{ \exp\left[2\pi \mathrm{i}k \left(\frac{\mu_{j'} + \xi}{\hbar\omega_c} - \frac{1}{2} \right) \right] \right\} \Big/ (\varepsilon - \xi - X_{j'}(\varepsilon)). \tag{11}$$

When $k \neq 0$, the range $|\xi| \ll \mu_j$ gives the main contribution to these integrals. Hence they may be taken along the whole real axis and one may use the residue theorem. The integration path has to be closed in the upper half-plane of the complex variable ξ for $k > 0$ and in the lower one for $k < 0$. When the equations (2) and (3) are satisfied, the integrals for $k = 0$ do not depend on $\hbar\omega_c$. They determine the Green function in the absence of a magnetic field. The imaginary parts of these integrals are calculated similarly to the ones for $k \neq 0$. The real parts, diverging at $\xi \to \infty$, determine the value of the chemical potential and can be assumed to be included into μ_j. Thus after integrating we obtain

$$X_j(\varepsilon) = \sum_{j'=1}^{M} \frac{\mathrm{i}\hbar}{2\tau_{jj'}} \operatorname{sign}[\operatorname{Im} X_{j'}(\varepsilon)] \times \left\{ 1 + 2 \sum_{k=1}^{\infty} \exp\left[-2\pi \mathrm{i}k \left(\frac{\mu_{j'} + \varepsilon - X_{j'}(\varepsilon)}{\hbar\omega_c} - \frac{1}{2} \right) \operatorname{sign}[\operatorname{Im} X_{j'}(\varepsilon)] \right] \right\}. \tag{12}$$

The signs of the imaginary parts, $\operatorname{sign}[\operatorname{Im} X_j(\varepsilon)]$, can be obtained from the limiting case $\tau_{jj'} \to \infty$ in zero magnetic field like for the 3D system [14]. In the weak-scattering limit, $X_j(\varepsilon) \to -\mathrm{i}0 \operatorname{sign}\varepsilon$. So we derive finally

$$X_j(\varepsilon) = -\sum_{j'=1}^{M} \frac{\mathrm{i}\hbar}{2\tau_{jj'}} \left\{ 1 + 2 \sum_{k=1}^{\infty} \exp[2\pi \mathrm{i}k F_{j'}(\varepsilon)] \right\} \operatorname{sign}\varepsilon \tag{13}$$

where

$$F_j(\varepsilon) = \left[\frac{\mu_j + \varepsilon - X_j(\varepsilon)}{\hbar\omega_c} - \frac{1}{2} \right] \operatorname{sign}\varepsilon. \tag{14}$$

It is seen from (13), (14) that at zero magnetic field

$$X_j(\varepsilon) = -\frac{\mathrm{i}\hbar}{2\tau_j} \operatorname{sign}\varepsilon. \tag{15}$$

In first order in the parameters $\exp(-\pi/\omega_c\tau_j)$, the solution of the system (13) has the form

$$X_j(\varepsilon) = -\sum_{j'=1}^{M} \frac{\mathrm{i}\hbar}{2\tau_{jj'}} \left\{ 1 + 2\exp(-\pi/\omega_c\tau_{j'}) \exp\left[2\pi \mathrm{i} \left(\frac{\mu_{j'} + \varepsilon}{\hbar\omega_c} - \frac{1}{2} \right) \operatorname{sign}\varepsilon \right] \right\} \operatorname{sign}\varepsilon. \tag{16}$$

One can see that the self-energy parts oscillate in a magnetic field. Equation (16) shows that the intersubband scattering rates become different from $1/\tau_{jj'}$ in the presence of a magnetic field due to small magneto-oscillations of the density of states in the j'th subband. For the same reason the scattering rate for the intersubband transition from the jth to the j'th subband differs from that for the reciprocal process.

Depending on sign ε, the Green function is analytical in the upper or lower half-planes of the complex variable ε. For this reason, one uses conveniently the advanced, G_j^A, and the retarded, G_j^R, Green functions as in the absence of a magnetic field:

$$G_j(\varepsilon,\xi) = \begin{cases} G_j^R(\varepsilon,\xi) & \varepsilon > 0 \\ G_j^A(\varepsilon,\xi) & \varepsilon < 0\,. \end{cases} \tag{17}$$

2.2. Finite temperature

To calculate the Green function and the conductivity tensor at non-zero temperature we will use the Matsubara technique. It is more convenient than the Keldysh technique because it allows one to obtain expressions for all components of the conductivity tensor in the same manner.

We go from a continuous variable, ε, to a discrete energy, $\mathrm{i}\epsilon_m$:

$$\epsilon_m = \pi T(2m+1) \tag{18}$$

where T is a temperature expressed in energy units and m is an integer. The temperature Green function has the form

$$\mathcal{G}_j(\epsilon_m,\xi_{j,n}) = [\mathrm{i}\epsilon_m - \xi_{j,n} - \mathcal{X}_j(\epsilon_m)]^{-1}. \tag{19}$$

The self-energy parts, $\mathcal{X}_j(\epsilon_m)$, satisfy a system of equations similar to (13):

$$\mathcal{X}_j(\epsilon_m) = -\sum_{j'=1}^{M} \frac{\mathrm{i}\hbar}{2\tau_{jj'}} \left\{ 1 + 2\sum_{k=1}^{\infty} \exp[2\pi \mathrm{i} k \Phi_{j'}(\epsilon_m)] \right\} \operatorname{sign} m \tag{20}$$

where

$$\Phi_j(\epsilon_m) = \left[\frac{\mu_j - \mathcal{X}_j(\epsilon_m)}{\hbar\omega_c} - \frac{1}{2} \right] \operatorname{sign} m + \frac{\mathrm{i}|\epsilon_m|}{\hbar\omega_c}. \tag{21}$$

To study the temperature behaviour of the conductivity, one has to obtain the values $\mathcal{X}_j(\epsilon_m)$ in second order in $\exp(-\pi/\omega_c\tau_j)$. With this accuracy, the solution of the system (20) takes the form

$$\begin{aligned} \mathcal{X}_j(\epsilon_m) = -\sum_{j'=1}^{M} \frac{\mathrm{i}\hbar}{2\tau_{jj'}} \Bigg\{ & 1 + 2\mathrm{e}^{-\pi/\omega_c\tau_{j'}} \exp\left[2\pi\mathrm{i}\left(\frac{\mu_{j'}}{\hbar\omega_c} - \frac{1}{2} \right) \operatorname{sign} m \right] \mathrm{e}^{-2\pi|\epsilon_m|/\hbar\omega_c} \\ & + \left(1 - \frac{2\pi}{\omega_c\tau_{j'}} \right) 2\mathrm{e}^{-2\pi/\omega_c\tau_{j'}} \exp\left[4\pi\mathrm{i}\left(\frac{\mu_{j'}}{\hbar\omega_c} - \frac{1}{2} \right) \operatorname{sign} m \right] \mathrm{e}^{-4\pi|\epsilon_m|/\hbar\omega_c} \Bigg\} \operatorname{sign} m. \end{aligned} \tag{22}$$

3. Conductivity tensor calculation

3.1. Conductivity at zero temperature

To calculate the conductivity tensor at $T = 0$, we will use the relationship (see appendix A) for an electric field frequency $\omega > 0$:

$$\sigma_{\alpha\beta}(\omega) = \frac{1}{\omega} \int\int \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{r}' \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} [\hat{J}_\alpha(\boldsymbol{r}) G_{\varepsilon+\hbar\omega}(\boldsymbol{r},\boldsymbol{r}')][\hat{J}_\beta(\boldsymbol{r}') G_\varepsilon(\boldsymbol{r}',\boldsymbol{r})] + \frac{\mathrm{i}Ne^2}{m\omega}\delta_{\alpha,\beta} \tag{23}$$

where the current-density operator has the form

$$\hat{\boldsymbol{J}}(\boldsymbol{r}) = \frac{e}{m}\left[-\mathrm{i}\hbar\,\boldsymbol{\nabla} - \frac{e}{c}\boldsymbol{A}(\boldsymbol{r}) \right]. \tag{24}$$

Here $\boldsymbol{A}(\boldsymbol{r})$ is the vector potential of the constant magnetic field applied perpendicular to the electron gas plane, N is the total 2D electron concentration, c is the velocity of light, e and m are the electron charge and effective mass respectively, $\delta_{\alpha,\beta}$ is the Kroneker symbol, α and β are Cartesian coordinates.

Below we will investigate the static conductivity and therefore consider the frequency ω as a small value and reduce it to zero in the final expressions. As the components of the conductivity tensor σ_{xx} and σ_{xy} are real at $\omega = 0$, one may calculate the value

$$\sigma = \sigma_{xx} + \mathrm{i}\sigma_{xy}.$$

Using the expression (5), calculating matrix elements of the current-density operator

$$\begin{aligned}\langle j'n'k'_y|\hat{J}_x|jnk_y\rangle &= -\mathrm{i}e\sqrt{\frac{\hbar\omega_c}{2m}}(\sqrt{n}\delta_{n',n-1} - \sqrt{n+1}\delta_{n',n+1})\delta_{k_y,k'_y}\delta_{j,j'} \\ \langle j'n'k'_y|\hat{J}_y|jnk_y\rangle &= -e\sqrt{\frac{\hbar\omega_c}{2m}}(\sqrt{n}\delta_{n',n-1} + \sqrt{n+1}\delta_{n',n+1})\delta_{k_y,k'_y}\delta_{j,j'}\end{aligned} \tag{25}$$

and taking into account degeneracy of a Landau level:

$$\sum_{k_y} = \frac{m\omega_c}{2\pi\hbar}$$

we obtain

$$\sigma(\omega) = \frac{e^2\omega_c^2}{2\pi\omega}\sum_{j=1}^{M}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{\mathrm{d}\varepsilon}{2\pi}\, nG_j(\varepsilon+\hbar\omega,\xi_{j,n})G_j(\varepsilon,\xi_{j,n-1}) + \frac{\mathrm{i}Ne^2}{m\omega}. \tag{26}$$

Equation (26) shows that all subbands give quasi-independent contributions to the conductivity. The probability of intersubband scattering enters into the values X_j only.

Applying the Poisson summation formula (10) and expressing $G_j(\varepsilon,\xi)$ through the advanced and retarded Green functions (17), we get

$$\begin{aligned}\sigma(\omega) = \frac{\mathrm{i}Ne^2}{m\omega} + \frac{e^2}{2\pi\hbar^2\omega}\sum_{j=1}^{M}\sum_{k=-\infty}^{\infty}\int_{-\mu_j}^{\infty}\mathrm{d}\xi\left(\mu_j - \frac{\hbar\omega_c}{2} + \xi\right)\exp\left[2\pi\mathrm{i}k\left(\frac{\mu_j+\xi}{\hbar\omega_c} - \frac{1}{2}\right)\right] \\ \times\left[\int_{-\infty}^{-\hbar\omega}\frac{\mathrm{d}\varepsilon}{2\pi}\, G_j^A(\varepsilon+\hbar\omega,\xi)G_j^A(\varepsilon,\xi-\hbar\omega_c)\right. \qquad (27a) \\ + \int_{-\hbar\omega}^{0}\frac{\mathrm{d}\varepsilon}{2\pi}\, G_j^R(\varepsilon+\hbar\omega,\xi)G_j^A(\varepsilon,\xi-\hbar\omega_c) \qquad (27b) \\ \left. + \int_{0}^{\infty}\frac{\mathrm{d}\varepsilon}{2\pi}\, G_j^R(\varepsilon+\hbar\omega,\xi)G_j^R(\varepsilon,\xi-\hbar\omega_c)\right]. \qquad (27c)\end{aligned}$$

Let us consider these calculations in detail. The terms in (27a) and (27c) with $k = 0$ give the magnetic field-independent contribution (see appendix B)

$$\frac{e^2}{2\pi\mathrm{i}\hbar^2\omega}\sum_{j=1}^{M}\mu_j$$

which cancels the 'gauge' part $\mathrm{i}Ne^2/m\omega$. This results from the absence of change of the carrier concentration in a magnetic field at the fixed chemical potential (see appendix C):

$$N = \sum_{j=1}^{M}N_j = \frac{m}{2\pi\hbar^2}\sum_{j=1}^{M}\mu_j. \tag{28}$$

Clearly, if the 2D concentration is fixed, then the chemical potential does not oscillate in a magnetic field either.

The rest of the terms in (27*a*) and (27*c*) and all of the terms in (27*b*) are determined by the range $|\xi| \ll \mu_j$. For this reason, the integration over ξ may be extended over the whole real axis and one can use the residue theorem. As the result we have

$$\sigma(\omega) = \frac{\mathrm{i}e^2}{2\pi\hbar^2\omega}\sum_{j=1}^{M}\left\{\int_{-\infty}^{-\hbar\omega}\mathrm{d}\varepsilon\left(\frac{\mu_j+\varepsilon}{\hbar\omega_c}\right)\sum_{k=1}^{\infty}[\mathrm{e}^{2\pi\mathrm{i}kF_j(\varepsilon+\hbar\omega)}-\mathrm{e}^{2\pi\mathrm{i}kF_j(\varepsilon)}]\right.$$
$$-\int_{-\hbar\omega}^{0}\mathrm{d}\varepsilon\,\frac{\mu_j}{\hbar\omega_c+X_j(\varepsilon+\hbar\omega)-X_j(\varepsilon)}\left[1+\sum_{k=1}^{\infty}(\mathrm{e}^{2\pi\mathrm{i}kF_j(\varepsilon+\hbar\omega)}-\mathrm{e}^{2\pi\mathrm{i}kF_j(\varepsilon)})\right]$$
$$\left.-\int_{0}^{\infty}\mathrm{d}\varepsilon\left(\frac{\mu_j+\varepsilon}{\hbar\omega_c}\right)\sum_{k=1}^{\infty}[\mathrm{e}^{2\pi\mathrm{i}kF_j(\varepsilon+\hbar\omega)}-\mathrm{e}^{2\pi\mathrm{i}kF_j(\varepsilon)}]\right\}. \tag{29}$$

In the second group of the integrals, we have taken it into account that the functions $X_j(\varepsilon)$ have breaks equal to $\mathrm{i}\hbar/\tau_j$ at $\varepsilon = 0$, and that therefore the differences $X_j(\varepsilon+\hbar\omega) - X_j(\varepsilon)$ are comparable with $\hbar\omega_c$. It should be noted that at $\omega \to 0$ the first and the third group of the integrals are imaginary, i.e. contribute to σ_{xy} only. They are equal to zero in the absence of a magnetic field.

Equation (29) determines the conductivity in all orders in $\exp(-\pi/\omega_c\tau_j)$. Substituting the self-energy parts (16) into (29), retaining the first-order terms in $\exp(-\pi/\omega_c\tau_j)$, and passing to the limit $\omega \to 0$, we get

$$\sigma_{xx} = \sum_{j=1}^{M}\frac{N_j e^2\tau_j/m}{1+(\omega_c\tau_j)^2}\left\{1+\frac{2(\omega_c\tau_j)^2}{1+(\omega_c\tau_j)^2}\delta_j+\sum_{j'\neq j}\frac{\tau_j}{\tau_{jj'}}\frac{1-(\omega_c\tau_j)^2}{1+(\omega_c\tau_j)^2}(\delta_j-\delta_{j'})\right\} \tag{30}$$

$$\sigma_{xy} = -\sum_{j=1}^{M}\frac{N_j e^2\tau_j^2\omega_c/m}{1+(\omega_c\tau_j)^2}\left\{1-\frac{1+3(\omega_c\tau_j)^2}{(\omega_c\tau_j)^2[1+(\omega_c\tau_j)^2]}\delta_j+\sum_{j'\neq j}\frac{\tau_j}{\tau_{jj'}}\frac{2}{1+(\omega_c\tau_j)^2}(\delta_j-\delta_{j'})\right\} \tag{31}$$

where

$$\delta_j = 2\cos\left(2\pi\frac{\mu_j}{\hbar\omega_c}+\pi\right)\exp\left(-\frac{\pi}{\omega_c\tau_j}\right). \tag{32}$$

Calculating the first integral group in (29), we put

$$\lim_{\varepsilon\to\infty}\exp(2\pi\mathrm{i}\varepsilon/\hbar\omega_c)$$

equal to zero. This is true at any finite temperature, when $\mathrm{i}\varepsilon$ is replaced by $-|\epsilon_m|$ and $m \to \infty$. Clearly, having calculated this limit, one can come back to zero temperature.

Equations (30)–(32) describe the conductivity of a quasi-2D system at zero temperature.

3.2. Conductivity at finite temperatures

At non-zero temperature, in the framework of the Matsubara technique, the conductivity is found at an imaginary frequency [14], $\mathrm{i}\omega_l$, where l is an integer and

$$\hbar\omega_l = 2\pi Tl.$$

Making in (29) the replacements

$$\omega \to \mathrm{i}\omega_l \qquad \varepsilon \to \mathrm{i}\epsilon_m \qquad X(\varepsilon) \to \mathcal{X}(\epsilon_m) \qquad \int \mathrm{d}\varepsilon \to 2\pi\mathrm{i}T\sum_{\epsilon_m} \tag{33}$$

we derive

$$\sigma(\mathrm{i}\omega_l) = \frac{\mathrm{i}e^2T}{\hbar^2\omega_l}\sum_{j=1}^{M}\Bigg\{\sum_{m=-\infty}^{-l-1}\left(\frac{\mu_j+\mathrm{i}\epsilon_m}{\hbar\omega_c}\right)\sum_{k=1}^{\infty}[\mathrm{e}^{2\pi\mathrm{i}k\Phi_j(\epsilon_{m+l})}-\mathrm{e}^{2\pi\mathrm{i}k\Phi_j(\epsilon_m)}]$$
$$-\sum_{m=-l}^{-1}\frac{\mu_j}{\hbar\omega_c+\mathcal{X}_j(\epsilon_{m+l})-\mathcal{X}_j(\epsilon_m)}\left[1+\sum_{k=1}^{\infty}(\mathrm{e}^{2\pi\mathrm{i}k\Phi_j(\epsilon_{m+l})}-\mathrm{e}^{2\pi\mathrm{i}k\Phi_j(\epsilon_m)})\right]$$
$$-\sum_{m=0}^{\infty}\left(\frac{\mu_j+\mathrm{i}\epsilon_m}{\hbar\omega_c}\right)\sum_{k=1}^{\infty}[\mathrm{e}^{2\pi\mathrm{i}k\Phi_j(\epsilon_{m+l})}-\mathrm{e}^{2\pi\mathrm{i}k\Phi_j(\epsilon_m)}]\Bigg\}. \tag{34}$$

When $|\epsilon_m| > \hbar\omega_c$, the Φ_j are small enough and we can neglect $\mathrm{i}\epsilon_m$ in comparison with μ_j. Expanding (34) up to the first order in $\exp(-\pi/\omega_c\tau_j)$ and summing the geometrical series, we will have the results (30) and (31) in which δ_j should be changed as follows:

$$\delta_j(T) = 2\cos\left(2\pi\frac{\mu_j}{\hbar\omega_c}+\pi\right)\exp\left(-\frac{\pi}{\omega_c\tau_j}\right)\frac{\lambda}{\sinh\lambda} \tag{35}$$

$$\lambda = \frac{2\pi^2T}{\hbar\omega_c}. \tag{36}$$

It follows that, in the first order in $\exp(-\pi/\omega_c\tau_j)$, the oscillations are damped out exponentially as the temperature increases. This damping is due to the temperature spreading of the electron distribution. It takes place when the temperature becomes larger than the energy distance between Landau levels.

In second order in $\exp(-\pi/\omega_c\tau_j)$, one obtains the terms $\exp[2\pi\mathrm{i}(\Phi_j(\epsilon_{m+l})+\Phi_{j'}(\epsilon_m))]$ in (34). It is important that some of them do not contain ϵ_m. The period of these oscillations in a reciprocal magnetic field is proportional to the energy distances between the subbands, $\mu_j-\mu_{j'}$. These amplitudes are not thermally damped out. There are oscillation terms, which become dominant as temperature increases. The expression for these non-damping terms has the form

$$\sigma_{xx}^{(nd)} = \sum_{j=1}^{M}\frac{N_je^2\tau_j/m}{[1+(\omega_c\tau_j)^2]^2}\left[\frac{2\tau_j}{\tau_{jj}}-1+(\omega_c\tau_j)^2\left(1-\frac{6\tau_j}{\tau_{jj}}\right)\right]$$
$$\times\sum_{j'\neq j}\frac{\tau_j}{\tau_{jj'}}2\cos\left(2\pi\frac{\mu_j-\mu_{j'}}{\hbar\omega_c}\right)\exp\left[-\frac{\pi}{\omega_c}\left(\frac{1}{\tau_j}+\frac{1}{\tau_{j'}}\right)\right] \tag{37}$$

$$\sigma_{xy}^{(nd)} = -\sum_{j=1}^{M}\omega_c\tau_j\frac{N_je^2\tau_j/m}{1+(\omega_c\tau_j)^2}\left\{1+\frac{2\tau_j}{\tau_{jj}}\frac{3-(\omega_c\tau_j)^2}{[1+(\omega_c\tau_j)^2]^2}\right\}$$
$$\times\sum_{j'\neq j}\frac{\tau_j}{\tau_{jj'}}2\cos\left(2\pi\frac{\mu_j-\mu_{j'}}{\hbar\omega_c}\right)\exp\left[-\frac{\pi}{\omega_c}\left(\frac{1}{\tau_j}+\frac{1}{\tau_{j'}}\right)\right]. \tag{38}$$

Note that in reference [7], some mistakes were made in expanding in the small parameter $\exp(-\pi/\omega_c\tau_j)$ and hence an incorrect result was obtained for $\sigma_{xx}^{(nd)}$. The component σ_{xy} was not calculated in reference [7].

The total expression for the conductivity, correct to $\exp(-2\pi/\omega_c\tau_j)$ order, can be presented as

$$\sigma = \sigma^{(0)}+\sigma^{(1)}+\sigma^{(2)}+\sigma^{(nd)}$$

where $\sigma^{(0)}$ and $\sigma^{(1)}$ are the non-oscillating and oscillating parts of (30) and (31) with δ_j from (35), and $\sigma^{(2)}$ is the second-order correction in $\exp(-\pi/\omega_c\tau_j)$, damping out with

temperature as $2\lambda/\sinh 2\lambda$. At zero temperature, $\sigma^{(2)}$ and $\sigma^{(nd)}$ are much smaller than $\sigma^{(1)}$. With increasing temperature, $\sigma^{(1)}$ and $\sigma^{(2)}$ decrease, and the latter more rapidly. For this reason, $\sigma^{(2)}$ should be neglected in comparison with $\sigma^{(1)}$ at any temperature. Thus the conductivity tensor is

$$\sigma_{\alpha\beta} = \sigma_{\alpha\beta}^{(0)} + \sigma_{\alpha\beta}^{(1)} + \sigma_{\alpha\beta}^{(nd)} \tag{39}$$

where the sum of the first and second terms is defined by (30) and (31) with δ_j from (35), and $\sigma_{\alpha\beta}^{(nd)}$ is given by (37), (38).

Note that in the framework of the theory developed, the chemical potential was considered to be fixed. However, it is possible to demonstrate that the expressions for $\sigma_{\alpha\beta}^{(1)}$ and $\sigma_{\alpha\beta}^{(nd)}$ have the same form in the case where the total carrier concentration is kept constant. The reason is that the oscillation correction to the chemical potential at fixed concentration is small in the parameter $(\hbar\omega_c/\mu)\exp(-\pi/\omega_c\tau)$ and, besides, shows exponential temperature damping [7].

4. Discussion

For the sake of definiteness, we will consider the magnetoconductivity oscillations for two-subband filling. Figure 1 and figure 2 demonstrate the magnetic field dependences of the resistance:

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

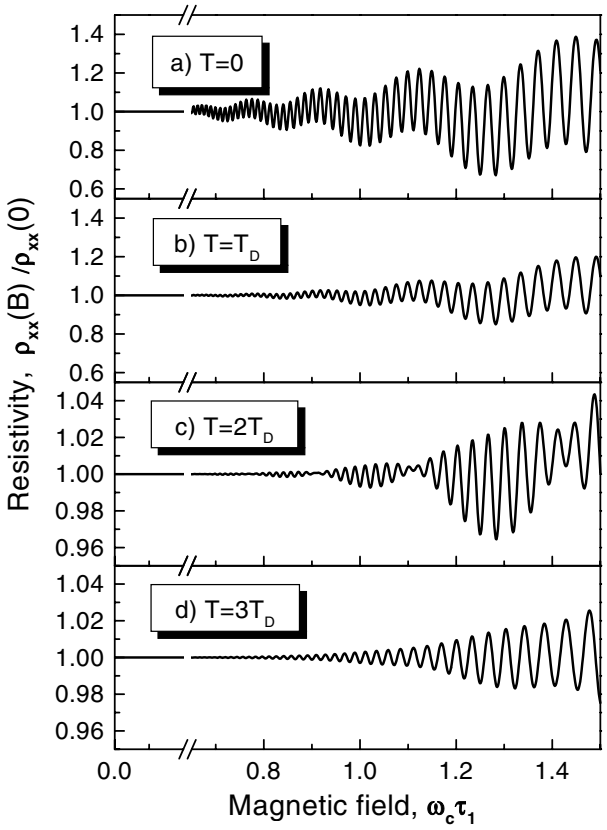

Figure 1. The temperature behaviour of the Shubnikov–de Haas oscillations for intense intersubband scattering, $\tau_{12} = 2\tau_1$.

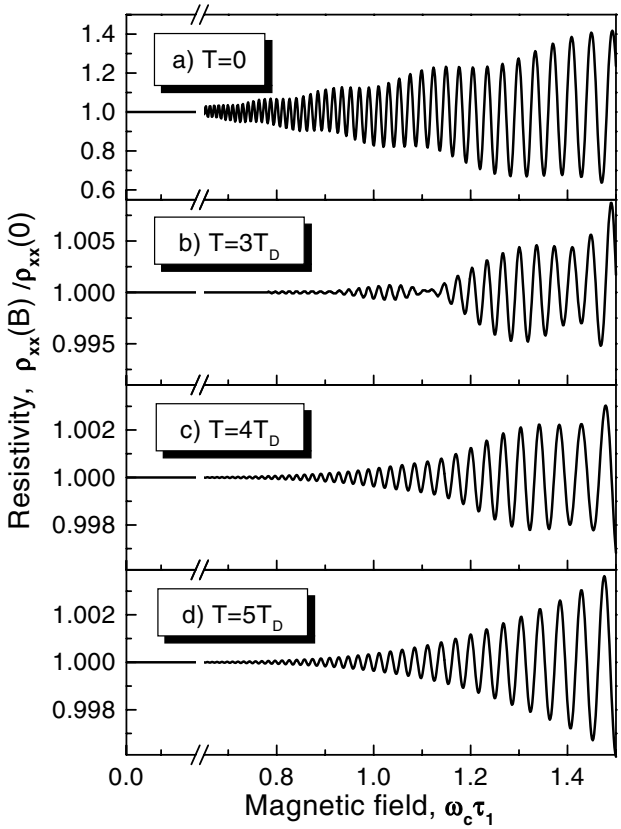

Figure 2. The temperature behaviour of the Shubnikov–de Haas oscillations for weak intersubband scattering, $\tau_{12} = 20\tau_1$.

calculated at different temperatures with the help of (39). The parameters used are the following: $\mu_1\tau_1/\hbar = 50$, $\mu_2\tau_2/\hbar = 5$, $\tau_1 = \tau_2$. We chose the filling of the excited subband to be relatively small because this case is often found in experiments on multilevel systems [15, 16]. Figure 1 presents the case of intense intersubband scattering ($\tau_1/\tau_{12} = 0.5$) and figure 2 corresponds to that of weak intersubband scattering ($\tau_1/\tau_{12} = 0.05$).

According to (30) and (31), the peculiarity of the quasi-2D Shubnikov–de Haas effect at low temperatures is as follows. For intense intersubband scattering, the oscillations $\cos(2\pi\mu_2/\hbar\omega_c)$ exist even at relatively small electron concentration in the excited subband (see figure 1(a)). This is because the scattering probability from the ground subband oscillates with two periods. Both low- and high-frequency harmonics are seen clearly (figure 1(a)). For weak intersubband scattering (figure 2(a)), the low-frequency oscillations may arise due to filling of the excited subband only and thus have relatively small amplitude ($N_2/N_1 = 0.1$). The Shubnikov–de Haas effect at low temperature was studied experimentally by de Lange [6]. Both the oscillations $\cos(2\pi\mu_1/\hbar\omega_c)$ and $\cos(2\pi\mu_2/\hbar\omega_c)$ were observed in accordance with our calculations at $T = 0$.

The other peculiarity of the quasi-2D system is the existence of oscillations which are not damped out thermally. These oscillations are of order $\exp(-2\pi/\omega_c\tau)$ and therefore are not observed at low temperatures. According to equation (35), the amplitudes of the main harmonics, $\sigma^{(1)}$ (and hence the oscillation amplitudes of σ) decrease while the temperature

increases from zero to $T = T_D$ (see figure 1(b)). Here

$$T_D = \frac{\hbar}{2\pi\tau_1}$$

is the Dingle temperature. As the temperature increases further (figures 1(c), 1(d)), the amplitudes of the ρ_{xx}-oscillations reduce non-exponentially. This indicates that there are terms, $\sigma^{(nd)}$, which are not damped out thermally. As seen from figure 1(c), the amplitudes $\sigma^{(1)}$ and $\sigma^{(nd)}$ are comparable at $T = 2\ T_D$. Since the excited subband is not filled much, the frequencies corresponding to the ground subband and the differential one differ by only 10% ($\mu_2/\mu_1 = 0.1$). Therefore a new effect, beats of the magnetoconductivity, appears (figure 1(c)). At $T = 3\ T_D$ (figure 1(d)), $\sigma^{(nd)}$ dominates, and hence the amplitudes and the shape of the magnetoresistance do not change as the temperature increases further.

If the intersubband scattering is weak, the role of the excited subband is not significant at zero temperature (figure 2(a)). With increasing temperature, this role is substantially enhanced, but the enhancement takes place at higher temperatures than in the case of intense intersubband scattering. The beats arise at $T = 3\ T_D$ (see figure 2(b)) for the parameter set listed above. At higher temperatures, the contribution $\sigma^{(nd)}$ begins to dominate and the oscillations $\sigma^{(1)}$ manifest themselves at the stronger magnetic fields only, when the ratio

$$\frac{\sigma^{(nd)}}{\sigma^{(1)}} \sim \frac{\tau_1}{\tau_{12}} \frac{T_D}{2\pi T} \exp\left[\frac{\pi}{\omega_c\tau_1}\left(\frac{T}{T_D} - 1\right)\right]$$

becomes of the order of unity. For instance, at $T = 4\ T_D$, $\sigma^{(1)}$ starts to play a significant role at $\omega_c\tau_1 > 1.3$ (figure 2(c)). At $T = 5\ T_D$, $\sigma^{(nd)}$ dominates over the whole range of magnetic fields shown (figure 2(d)).

Experimentally, the temperature dependence of the magneto-oscillations was studied in references [4, 5, 16]. These results agree qualitatively with the calculations given. The results [16] agree better with figure 2 than figure 1. This implies that the intersubband scattering was weak in the samples investigated. Quantitatively this confirms the large—by more than two orders of magnitude—reduction of the oscillation amplitude, which can take place only when the ratio τ_1/τ_{12} is small (see (37), (38)).

5. Conclusions

In this paper, the general theory of the magneto-oscillation effects was developed for quasi-2D semiconductor systems. The magnetoresistance tensor has been calculated for arbitrary number of occupied size-quantized subbands. For the first time, the temperature dependence of the Shubnikov–de Haas oscillations was analysed in detail. It is demonstrated that, at any intensity of the intersubband scattering, this process is responsible for the magnetoconductivity behaviour with increasing temperature. It was predicted that a new effect of emerging and damping out of the magnetoconductivity beats occur in the case of relatively small filling of the upper subband. The theory developed agrees qualitatively with available experimental data.

Acknowledgments

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Appendix A

To calculate the conductivity tensor, $\sigma_{\alpha\beta}(\omega)$, in a magnetic field we apply to the system an external uniform electric field, $\boldsymbol{E}$, parallel to the 2D plane. Let $\delta\boldsymbol{A}(t)$ be the vector potential of this field:

$$\boldsymbol{E} = (-1/c)\,\partial\,\delta\boldsymbol{A}/\partial t.$$

The relationship between the Fourier transforms of the current density, $\boldsymbol{j}(\omega)$, and the electric field has the form

$$j_\alpha(\omega) = \frac{\mathrm{i}\omega}{c}\sigma_{\alpha\beta}(\omega)\,\delta A_\beta(\omega). \tag{A.1}$$

On the other hand, the current density may be directly expressed through the exact Green function

$$\boldsymbol{j}(\boldsymbol{r},t) = \left\{\frac{\mathrm{i}e}{2m}[\boldsymbol{p}(\boldsymbol{r}') - \boldsymbol{p}(\boldsymbol{r})] + \frac{\mathrm{i}e^2}{mc}(\boldsymbol{A}(\boldsymbol{r}) + \delta\boldsymbol{A}(t))\right\}_{\boldsymbol{r}'\to\boldsymbol{r}} \tilde{G}(\boldsymbol{r},t;\boldsymbol{r}',t+0) \tag{A.2}$$

where $\boldsymbol{A}(\boldsymbol{r})$ is a vector potential of the homogeneous magnetic field, $\boldsymbol{p} = -\mathrm{i}\,\boldsymbol{\nabla}$ is the momentum operator, and $\tilde{G}(\boldsymbol{r},t;\boldsymbol{r}',t')$ is the Green function in the field $\boldsymbol{A}+\delta\boldsymbol{A}$. Expanding $\tilde{G}(\boldsymbol{r},t;\boldsymbol{r}',t')$ up to terms linear in $\delta\boldsymbol{A}$, we get from (A.2) [12]

$$\begin{aligned} j_\alpha(\boldsymbol{r},t) = {} & \frac{\mathrm{i}e^2}{mc}G(\boldsymbol{r},t,\boldsymbol{r},t+0)\,\delta A_\alpha(t) + \frac{\mathrm{i}e^2}{m^2c}\left[\frac{p_\alpha(\boldsymbol{r}) - p_\alpha(\boldsymbol{r}')}{2} - \frac{e}{c}A_\alpha(\boldsymbol{r})\right]_{\boldsymbol{r}'\to\boldsymbol{r}} \\ & \times \int \mathrm{d}t_1 \int \mathrm{d}\boldsymbol{r}_1 \left[\frac{p_\beta(\boldsymbol{r}_1) - p_\beta(\boldsymbol{r}_1')}{2} - \frac{e}{c}A_\beta(\boldsymbol{r}_1)\right]_{\boldsymbol{r}_1'\to\boldsymbol{r}_1} \\ & \times G(\boldsymbol{r},t;\boldsymbol{r}_1',t_1)G(\boldsymbol{r}_1,t_1;\boldsymbol{r}',t)\delta A_\beta(t_1) \end{aligned} \tag{A.3}$$

where G is the Green function in the field $\boldsymbol{A}$. Passing to Fourier components $(\boldsymbol{r},t) \to (\boldsymbol{q},\omega)$ and taking into account (A.1), we obtain for $\boldsymbol{q} = 0$

$$\begin{aligned} \sigma_{\alpha\beta}(\omega) = {} & \frac{\mathrm{i}Ne^2}{m\omega}\delta_{\alpha,\beta} + \frac{e^2}{m^2\omega}\int\int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}_1 \int_{-\infty}^{\infty}\frac{\mathrm{d}\varepsilon}{2\pi}\left[\frac{p_\alpha(\boldsymbol{r}) - p_\alpha(\boldsymbol{r}')}{2} - \frac{e}{c}A_\alpha(\boldsymbol{r})\right]_{\boldsymbol{r}'\to\boldsymbol{r}} \\ & \times \left[\frac{p_\beta(\boldsymbol{r}_1) - p_\beta(\boldsymbol{r}_1')}{2} - \frac{e}{c}A_\beta(\boldsymbol{r}_1)\right]_{\boldsymbol{r}_1'\to\boldsymbol{r}_1} G_{\varepsilon+\hbar\omega}(\boldsymbol{r},\boldsymbol{r}_1')G_\varepsilon(\boldsymbol{r}_1,\boldsymbol{r}'). \end{aligned} \tag{A.4}$$

Here N is the total electron concentration (see (C.1)). Integrating (A.4) by parts, we get finally

$$\sigma_{\alpha\beta}(\omega) = \frac{1}{\omega}\int\int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}_1 \int_{-\infty}^{\infty}\frac{\mathrm{d}\varepsilon}{2\pi}\,[\hat{J}_\alpha(\boldsymbol{r})G_{\varepsilon+\hbar\omega}(\boldsymbol{r},\boldsymbol{r}_1)][\hat{J}_\beta(\boldsymbol{r}_1)G_\varepsilon(\boldsymbol{r}_1,\boldsymbol{r})] + \frac{\mathrm{i}Ne^2}{m\omega}\delta_{\alpha,\beta} \tag{A.5}$$

where the current-density operator is

$$\hat{\boldsymbol{J}}(\boldsymbol{r}) = \frac{e}{m}\left[-\mathrm{i}\hbar\,\boldsymbol{\nabla} - \frac{e}{c}\boldsymbol{A}(\boldsymbol{r})\right]. \tag{A.6}$$

Appendix B

To avoid the appearance of divergences in (27*a*) and (27*c*), the terms with $k = 0$ should be integrated over ε first, as for a simple system in the absence of a magnetic field [12]. However, in the case under study, the Green function contains the variable ε in the self-energy parts, $X_j(\varepsilon)$ (see (13)). To overcome this difficulty, we take it into account that

$$X_j(\varepsilon) = -\operatorname{sign}\varepsilon\,\frac{\mathrm{i}\hbar}{2\tau_j}[1 + \gamma_j(\varepsilon)] \tag{B.1}$$

where $|\gamma_j(\varepsilon)| \ll 1$ (see (16)). Since these parameters are small, the Green functions may be expanded in the series

$$G_j^{R,A}(\varepsilon,\xi) = \frac{1}{\varepsilon - \xi \pm \mathrm{i}\hbar/2\tau_j} + \sum_{l=1}^{\infty} \frac{(\mp \mathrm{i}\hbar/2\tau_j)^l [\gamma_j(\varepsilon)]^l}{[\varepsilon - \xi \pm \mathrm{i}\hbar/2\tau_j]^{l+1}}. \tag{B.2}$$

The first term depends on ε in a simple way. A product of two such terms in (27*a*) and (27*c*) can be easily integrated over ε and then over ξ. If one passes in the expression obtained to the limits $\mu_j/\hbar\omega_c \to \infty$, $\mu_j\tau_j/\hbar \to \infty$, then one gets the contribution

$$\frac{e^2}{2\pi \mathrm{i}\hbar^2\omega} \sum_{j=1}^{M} \mu_j. \tag{B.3}$$

The rest of the terms in (27*a*) and (27*c*) contain the parts of equations (B.2) with $l \geqslant 1$. In equation (27*c*), $\varepsilon > 0$, and the characteristic values of ξ are of the same order as ε. Hence one can perform the integration over ξ over the whole real axis. Since the integrals contain products of three or more Green functions with the same imaginary parts, they are equal to zero according to the residue theorem.

The integrals (27*a*) can be conveniently transformed as follows:

$$\int_{-\infty}^{-\hbar\omega} \mathrm{d}\varepsilon \int_{-\mu_j}^{\infty} \mathrm{d}\xi = \int_{-\infty}^{-\hbar\omega} \mathrm{d}\varepsilon \int_{-\infty}^{\infty} \mathrm{d}\xi - \int_{-\infty}^{-\mu_j} \mathrm{d}\xi \int_{-\infty}^{-\hbar\omega} \mathrm{d}\varepsilon. \tag{B.4}$$

The first integral here is equal to zero according to the residue theorem. In the second one, the characteristic $\varepsilon \sim \xi < -\mu_j$. Therefore one can extend the integration over ε to the whole real axis. There are one or more factors $\gamma_j(\varepsilon)$ in the integral. They depend on ε as follows:

$$\gamma_j(\varepsilon) \sim \exp(-2\pi \mathrm{i} s\varepsilon/\hbar\omega_c) \tag{B.5}$$

where s is a positive integer (see (16)). Therefore one can close the integration path in the lower half-plane of the complex variable ε. There, the functions $(\varepsilon - \xi - \mathrm{i}\hbar/2\tau_j)^{-(l+1)}$ are analytical. Hence these integrals are equal to zero as well.

Thus the terms with $k = 0$ in (27*a*) and (27*c*) give the contribution (B.3).

Appendix C

The electron concentration, N, may be expressed via the Green function [14]:

$$N = -\mathrm{i} \int \mathrm{d}\boldsymbol{r} \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi}\, G_\varepsilon(\boldsymbol{r},\boldsymbol{r}) \mathrm{e}^{\mathrm{i}\varepsilon 0}. \tag{C.1}$$

Representing the Green function in the form (5) and making use of the Poisson summation formula (10), one gets from equation (C.1)

$$N = -\mathrm{i}\frac{m}{2\pi\hbar^2} \sum_{j=1}^{M} \sum_{k=-\infty}^{\infty} \int_{-\mu_j}^{\infty} \mathrm{d}\xi \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi}\, \exp\left[2\pi \mathrm{i}k\left(\frac{\mu_j + \xi}{\hbar\omega_c} - \frac{1}{2}\right)\right] G_j(\varepsilon,\xi) \mathrm{e}^{\mathrm{i}\varepsilon 0}. \tag{C.2}$$

Expanding $G_j(\varepsilon,\xi)$ in the series (B.2) and detaching in (C.2) the term with $k = 0$, one can express the concentration as a sum of three groups of integrals:

$$\begin{aligned} N = -\mathrm{i}\frac{m}{2\pi\hbar^2} \sum_{j=1}^{M} \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} \Bigg\{ & \int_{-\mu_j}^{\infty} \mathrm{d}\xi\, \frac{\mathrm{e}^{\mathrm{i}\varepsilon 0}}{\varepsilon - \xi + (\mathrm{i}\hbar/2\tau_j)\operatorname{sign}\varepsilon} \\ & + \sum_{l=1}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}\xi\, \frac{(-\mathrm{i}\hbar/2\tau_j)^l (\operatorname{sign}\varepsilon)^l [\gamma_j(\varepsilon)]^l}{[\varepsilon - \xi + (\mathrm{i}\hbar/2\tau_j)\operatorname{sign}\varepsilon]^{l+1}} \\ & + \sum_{k\neq 0} \int_{-\infty}^{\infty} \mathrm{d}\xi\, \exp\left[2\pi \mathrm{i}k\left(\frac{\mu_j + \xi}{\hbar\omega_c} - \frac{1}{2}\right)\right] \Big/ (\varepsilon - \xi - X_j(\varepsilon)) \Bigg\}. \end{aligned} \tag{C.3}$$

The second group of integrals here is equal to zero according to the residue theorem. Calculating the rest, we get

$$N = \frac{m}{2\pi\hbar^2}\sum_{j=1}^{M}\left\{\int_{-\mu_j}^{\infty} \mathrm{d}\xi\ \frac{\mathrm{e}^{\mathrm{i}\xi 0}}{\pi}\left[\frac{\pi}{2} - \arctan(2\xi\tau_j/\hbar)\right] - \sum_{k=1}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}\varepsilon\ \mathrm{sign}\,\varepsilon\,\mathrm{e}^{2\pi\mathrm{i}kF_j(\varepsilon)}\right\}. \quad \text{(C.4)}$$

In the limit $\mu_j\tau_j/\hbar \to \infty$, the first integral is equal to μ_j, and the second one may be presented as

$$\hbar\omega_c \sum_{s=1}^{\infty} A_s \exp(-\pi s/\omega_c\tau_j)$$

where $|A_s| < 1$. The explicit form of the coefficients A_s, oscillating in a magnetic field, may be obtained by solving the Dyson equation (13). However, it is clear that at $\hbar\omega_c \ll \mu_j$ this contribution to the concentration is small. Therefore the concentration remains constant in this range of magnetic field:

$$N = \frac{m}{2\pi\hbar^2}\sum_{j=1}^{M}\mu_j. \quad \text{(C.5)}$$

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